

Leibniz's last controversy with the Newtonians.

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In this paper I shall give a critical account of the famous series of controversial letters which Leibniz exchanged at the end of his life with Dr. Samuel Clarke. In this correspondence Clarke is the very able representative of Newton and the Newtonians, and Leibniz is engaged in controverting certain fundamental points in the Newtonian philosophy of nature. Questions of pure mathematics, such as Fluxions and Differentials, play no important part in these letters.

I shall begin with a few elucidatory remarks about the persons concerned and the historical background of the controversy. Newton does not appear in person upon the scene, but Clarke is his deputy and advocate.

It will suffice to say a very few words by way of reminder about Newton and Leibniz, but it will be desirable to give a little more detail about Clarke. Newton was born on Old Christmas Day 1642; he entered Trinity College, Cambridge as a subsizar in June 1661 and became a Fellow of the College in 1667 and Lucasian Professor of Mathematics in Cambridge University in 1669; he published his *Philosophiae Naturalis Principia Mathematica* in the summer of 1687; he became Master of the Mint in 1699 and lived thereafter in London until his death on March 20th., 1727. He was 73 years old when Leibniz wrote the letter which opened this controversy.

Leibniz was born on January 21st., 1646, at Leipzig. He was

appointed Librarian to the Duke of Brunswick at Hanover in 1676, and he held that post until his death on November 14th. He was, to the best of my belief, the greatest pure intellect of whom we have any record. He was 69 years old when he started this controversy and he died in the midst of it.

Samuel Clarke was born at Norwich on Oct. 11th. 1675. Though not a man of genius, like Newton and Leibniz, he was a person of first-rate mathematical and philosophical ability. He entered Caius College, Cambridge, in 1691, and became a very distinguished undergraduate known to his contemporaries as 'the lad of Caius'. He soon became an enthusiastic adherent of the Newtonian system in opposition to that of Descartes. At that time one of the best text-books of mathematical physics was Rohault's *Physique*, written from the Cartesian standpoint. On the advice of his Tutor, Clarke translated it into Latin and provided it with copious critical notes in which its Cartesian errors were indicated and corrected on Newtonian principles. Clarke's translation of Rohault became the standard text-book in Cambridge. It was later translated into English by Clarke's brother John, and remained in use until at least 1730.

Clarke became a clergyman of the Church of England, like his father before him, and in 1709 was appointed Chaplain in Ordinary to Queen Anne. In that year he took his D. D. degree at Cambridge. In order to do this he defended before the University the thesis that no article of the Christian faith is contrary to reason. His academic opponent was Dr. James, the Regius Professor of Divinity. It was usual for the Professor on such occasions to conclude the exercise by saying to the candidate *Probe te exercui*; on this occasion Dr. James substituted the phrase *Probe me exercuisti*. A Dr. Yarborough, who had been present, said many years afterwards when he was an old clergyman living at a great distance: "Though I am 77 years old, I would gladly ride to Cambridge to hear such another act."

Clarke was probably the ablest philosophical theologian in England in the early XVIII-th. Century. He is famous for his attempt to found a purely rationalistic and axiomatic system of

ethics. His theological and ethical theories are stated with great force and clearness in the course of Boyle Lectures which he delivered in London in 1704'5. He was offered the Mastership of the Mint on Newton's death in 1727, but declined it as an unsuitable office for a clergyman. He himself died on May 17th., 1729.

There is another person to be mentioned, though she, like Newton, remains behind the scenes. This is Caroline of Anspach, at that time Princess of Wales and afterwards Queen of England as Consort to George II. She was the very intelligent and highly astute wife of a rather stupid and tiresome husband. She managed George admirably in his own interests; making no needless fuss over the regular mistresses whom he thought it due to this position to keep at home, or the occasional infidelities abroad which he was wont to describe in great detail in the eloquent and affectionate letters which he would write to her from Hanover; and getting her own way by proposing the opposite course and relying on his habit of automatically contradicting what he believed to be her wishes. Though Caroline was accustomed to call a spade a spade, and, to judge from Lord Hervey's *Memoirs*, had a fondness for highly spiced conversation, she also took a genuine delight in the company of philosophers and theologians, and seems to have been capable of following and appreciating their arguments. She had been friendly with Leibniz in Hanover; when she came to England after the death of Queen Anne she saw much of Clarke; and in her later years she conversed with Berkeley and befriended Butler, the two great philosophic bishops whose works are still read by philosophers throughout the western world.

The controversy between Leibniz and Clarke began with a letter written by the former to the Princess of Wales in Nov. 1715. In this letter Leibniz made some provocative remarks about the decay of religion and the growth of materialism in England, and attributed this to certain doctrines of Newton and of Locke which he specifies. Caroline handed the letter to Clarke who answered it in some detail. Leibniz replied, and a

general engagement along the whole philosophical line followed. Each party wrote five letters, of gradually increasing length and acerbity. Clarke may be said to have remained in possession of the field, since Leibniz died in Nov. 1716 before he could answer Clarke's fifth letter.

The central subject under discussion in these letters is the nature of Space and Time. Newton, as is well known, held a form of the Absolute Theory of Space and Time. This was because he thought that certain dynamical phenomena both enable and compell us to distinguish between absolute and relative rotation in particular and between absolute and relative acceleration in general. He held that absolute rotation and acceleration entail absolute space and time. Leibniz rejected the Absolute Theory and was one of the first persons to state the alternative Relational Theory clearly. His main grounds for rejecting the Absolute Theory were that it conflicts, in his opinion, with two general philosophical principles which he set great store by, viz., the Principle of Sufficient Reason and the Identity of Indiscernibles.

Closely connected with this central topic are certain subsidiary questions. Is the material universe limited or unlimited in extension? Is the created world limited or unlimited in duration? Are there empty regions within the material world, or is it a plenum? On all these questions the Newtonians held certain views and Leibniz was opposed to them.

Much of the controversy is conducted in theological terms. This is partly adventitious; many of the questions which are stated and discussed in theological terminology are independent of it and could easily be translated into non-theological language. But this is not true of all of them. Both Leibniz and the Newtonians were convinced theists, who took the notion of God as creator and sustainer of the world seriously, and there are many assertions about God and his operations in Newton's *Principia*. Newton had thrown out the suggestion that Absolute Space might be the *sensorium* of God, i. e., roughly speaking the medium in and through which God perceives created things.

Leibniz attacks this bitterly and somewhat tediously as leading to consequences which are theologically unacceptable. But it develops into the more general and interesting question: 'How does God perceive material things; how do men perceive them; and how are the two modes of perception inter-related?' On these questions Leibniz held highly original and rather paradoxical views which Clarke could not accept. Again, to any theist the question: 'How are Space and Time related to God?' is of fundamental importance; and it is plain that, whatever the answer may be, it will be very different according as the Absolute or the Relational theory is presupposed.

Two other central questions in the correspondence are the Newtonian Theory of Attraction and what I shall describe by an intentionally vague expression as the 'Conservation or Non-Conservation of Active Force'. Newton's theory of gravitation as a not further explicable property of matter seemed to Leibniz radically unscientific; it was for him a betrayal of the hardly and recently won principle that all genuine explanation of natural phenomena must be in mechanical terms, and a reversion to the purely verbal explanations of the mediaeval scholastic philosophers. On the second question the Newtonians held that the created universe is automatically running down; whilst Leibniz, on metaphysical grounds, had formulated and asserted what we should now call the 'Conservation of Vis Viva', and thought that this disproved the Newtonian contention. The discussion of both these subjects led straightway to the theological topic of the nature of miracles, i. e., the distinction between the natural and the super-natural action of God on the created world. Leibniz said that the only way to make sense of the Newtonian theory of attraction would be to suppose that, when one material particle is moving in the neighbourhood of another, God diverts the former from the straight line which it would otherwise traverse with uniform velocity in accordance with the Law of Inertia. This, he says, is to introduce a continual miracle into the ordinary course of nature. He also said that it is discreditable to the skill of God as maker of the world-machine to suppose

that he needs every now and then to clean it and wind it up miraculously.

I think that this should suffice to give a general idea of the contents of the correspondence. I shall now state the Newtonian view of Space and Time, as it gradually emerges in Clarke's letters under the stimulus of Leibniz's criticisms. I suspect that it was much vaguer in many respects when it left Newton's hands than it became when Clarke had to defend it against a critic of Leibniz's ability.

(1) The first and most fundamental point is that Space is logically prior to matter, and Time is logically prior to events or processes. There could not have been matter unless there had been Space for it to occupy and to rest or move in and Time for it to endure through. There could not have been events or processes unless there had been Time in which they have their dates and their durations. But there would have been Space, in precisely the same literal categorical sense, even if there had never been any matter; and there would have been Time, in precisely the same literal and categorical sense, even if there had never been any events or processes.

(2) We must distinguish between the space occupied by a body at any moment or for any period and the volume of that body; for the body could occupy different spaces at different times without changing its volume. The volume of a body is a property of it, but the space which it occupies at any time is not. Limited spaces are not properties of limited bodies, even if they happen to be occupied by such; they are just parts of the one unlimited Space in which these limited substances exist. Even if it were the case that the whole of infinite Space were continuously occupied by matter, still Space would not be a property of that infinite body. The infinite mass of matter would still merely be *in* Space as finite bodies are in it.

It is plain that Clarke takes a similar view, *mutatis mutandis*, about Time and events or processes in Time; though I think that he ought to have paid more attention to the fact that it is meaningless to suppose that an event or process should shift

its position in Time as a body can shift its position in Space without change of volume or shape.

(3) Strictly speaking Space is indivisible. One can indeed talk of parts of Space, i. e., different regions actually or in imagination marked out by containing certain material objects or by being traced in pencil or ink. But the parts of Space are in principle inseparable. Two adjoined regions of Space are inseparable, not merely in the sense that there is no force in nature which could overcome their mutual adhesion. This would be the case with two adjoined parts of an old-fashioned extended solid atom. But two adjoined regions of Space are inseparable in the logical sense that it is *meaningless* to talk of any region coming to occupy a different position, and it is therefore *meaningless* to suggest that two regions which are adjoined might be separated. In the same way Space is in principle continuous. It is a contingent question whether there are or are not holes in matter; it is nonsensical to suggest that there might be holes in Space. Similar remarks apply, *mutatis mutandis*, to Time.

(4) Space is actually, and not just potentially, infinite. Of course neither Clarke nor Leibniz ever entertained the notion that the geometry of nature might be non-Euclidean and that straight lines might return into themselves like the great-circles on a sphere. The same is true of Time; it had no beginning and will have no end.

(5) The points of Space and the moments of Time are not perceptible; only the things and events which occupy Space and Time can be perceived. But, since Time is quite independent of the events and processes which happen to occupy it, it is intelligible to suggest that the universe might have been created at an earlier or a later moment than that at which it was in fact created. Again, since Space is quite independent of the things and events which happen to occupy it, both the following suggestions are intelligible on the supposition that the material universe is of finite extent. (i) That without any difference in its internal structure it might *have been* created in a different

region of Space. (ii) That it might *be* moved as a whole by God from one part of Space to another, or be given an absolute rotation about any direction in Space. If this rectilinear motion of the universe were accelerated or decelerated, or if the universe were subjected to an absolute rotation, these absolute motions would betray themselves by observable forces within the world. Otherwise they would remain unobservable.

(6) Absolute motion involves absolute Space and absolute Time; and the existence of absolute motion and its distinction from relative motion is evidenced by the existence of centrifugal forces, by the flattening of the earth at the poles, and so on. Clarke points out two consequences of the theory that all motion is relative which are certainly most paradoxical and which seem to him enough to refute the theory. One is that, if a body happened to be the only one that existed, it would be meaningless to suggest that it could either be at rest or in motion whether translatory or rotational. The other is that, if all the matter outside a rotating body, such as the earth, were annihilated, it would at once become meaningless to say that it was rotating; and therefore presumably all the observable effects which are attributed to the rotation of the earth would cease to happen.

(7) A region of Space or a stretch of time has an absolute magnitude, viz. volume in the one case and duration in the other. Different regions can be compared in respect of their volumes, and different stretches in respect of their duration.

(8) The last topic which must be discussed here is Clarke's account of what might be called the 'ontological status' of Absolute Space and Time. The following are the main points. (i) They are not substances, but attributes. (ii) They are attributes, not of any created substance, but of God himself. Absolute Space is that attribute of God which theologians call 'Immensity'; Absolute Time is that attribute of God which they call 'Eternity'. Clarke says, somewhat rashly in my opinion, that no meaning can be attached to 'immensity' except space without bounds, and no meaning can be attached to 'eternity' except time without beginning or end. (iii) Absolute Space and Time are

said to be, not only attributes of God, but also immediate and necessary consequences of his existense. Since they are attributes which follow necessarily from the existence of a Being whose existence is necessary, their ontological status is much more assured than that of any material thing or event. For the existence of the latter is contingent, depending as it does on the will of God to create it. (iv) God does not 'exist in' Space and Time in the sense in which created things and events do so. For Space and Time are logically prior to created things and events, and, if a certain region of Space happens to be occupied for a certain stretch of time by a certain thing or process, that is simply because God chose to create such a thing or process at a certain place and date. Obviously God is not 'in' Space and Time, which are his own attributes, in this special way. Nevertheless God is immediately present throughout the whole of unending time to every part of unbounded space. By this omnipresence he is continually aware of all created things and he acts upon them, but they do not react upon him.

I pass now to Leibniz's criticisms on the Newtonian theory of Space and Time and to the alternative Relational Theory of them which he upheld in its stead. I shall begin with an account of his two principles of Sufficient Reason and the Identity of Indiscernibles.

Leibniz distinguished sharply between necessary and contingent truths. A truth is necessary if and only if all the apparent alternatives to it are impossible because self-contradictory. Thus, e. g., the proposition that the square-root of 2 is irrational is a necessary truth. For the supposition that there is a fraction m/n , in its lowest terms, such that $m^2 = 2n^2$ can be shown to be self-contradictory. A truth is contingent if and only if there are real alternatives to it which, though in fact false, are logically possible because internally consistent. Thus, e. g., it is a contingent singular truth that Julius Cæsar decided to cross the Rubicon on a certain occasion, and it is a contingent general truth that the sine of the angle of incidence bears a constant ratio to the sine of the angle of refraction for light of a given

wave-length travelling from a certain medium to a certain other.

It is clear that the Principle of Non-Contradiction is the guarantee of necessary truths, and it is equally clear that it is not the guarantee of contingent truths. Now Leibniz held that, in the case of any contingent truth, there is always a sufficient reason why that proposition is true and why the logically possible alternatives to it are false. He also held that the ultimate reason for the truth of any true contingent proposition is always of the same kind. If we trace this doctrine backwards in the letters to Clarke, we find that it rests on the following two interconnected principles. (i) Every choice is determined by motives. (ii) Any being who is capable of choosing always chooses that alternative which seems to him at the time to be the most good or the least bad of those open to him. In comparing alternatives from this point of view he will consider, not only the intrinsic qualities of each, but also its relations to contemporary and past events and its future consequences. He will choose that one which seems to him to be most good or least bad on the whole when all these factors are taken into account.

Now these general principles of choice apply to God as well as to created intelligent beings such as men or angels. But there are certain important differences between God and any created being in this matter. God is fully aware of all the possible alternatives, and can see all the relationships and foresee all the consequences of each. But a created being is always limited in the extent of his knowledge and is always liable to have mistaken beliefs about matters of fact. Moreover what seems best on the whole to God is always what is really best on the whole. But a creature is always liable to be biassed by passion or impulse, so that what seems best to him may not really be so even if he makes no mistakes or omissions about matters of fact.

Now the ultimate reason for the truth of any true contingent proposition is this. God foresaw that a world in which this proposition would be true would on the whole contain more good or less evil than any possible alternative world in which

it would be false. He therefore chose to create a world in which this proposition would be true, and to leave uncreated all the equally possible worlds in which this proposition would be false and one or other of the possible alternatives to it would have been true. What I have just been explaining is, I think, what Leibniz meant by the *Principle of Sufficient Reason*.

We come now to the other Principle, viz., the *Identity of Indiscernibles*. McTaggart used to say, rightly in my opinion, that a better name for the principle would be the *Dissimilarity of the Diverse*. Leibniz held that we can know for certain that there are not, never have been, and never will be two things in nature which are exactly alike. If there is numerical diversity, there is certainly some kind and degree of qualitative dissimilarity. He undoubtedly meant this much by the Identity of Indiscernibles. What is uncertain is whether he held that the very supposition that there might be two things exactly alike is self-contradictory and meaningless; or whether he held that, although it is not logically impossible that there should be two such things, we can be quite sure that God would not create them. As Clarke pointed out, Leibniz seems now to say the one thing and now the other. In his Fourth Letter, e. g., he says that 'to suppose two indiscernible things is to suppose the *same* thing under two names'. This certainly suggests that he held that the supposition, if taken literally, is self-contradictory and meaningless. But elsewhere in this Letter, and still more explicitly in the Fifth, he seems to take the other view. For instance, in the Fifth Letter he says that he does *not* maintain that it is absolutely impossible to suppose that there are two bodies which are indiscernible from each other, but only that it would be *contrary to God's wisdom* to create two such bodies and therefore we can be certain that there are not two such.

I think that there are two things to be said about this apparent inconsistency. (i) Plainly there is a sense in which it is possible to make and to argue correctly and intelligibly from a supposition which is, in another sense, impossible. That is precisely what happens, e. g., when one proves by a *reductio ad absurdum* that

there cannot be a rational fraction in its lowest terms whose square is equal to 2. (ii) Leibniz might merely be making a concession, for the sake of argument, to his opponent. His position might, perhaps, be expressed as follows: 'I can see that the supposition that there are two things exactly alike is self-contradictory; but, even if you will not grant me this, I can show from the Principle of Sufficient Reason that God never would create two such things and therefore that the supposition will always be false.'

We can now pass from the statement of Leibniz's two philosophical principles to the use which he makes of them in attacking the Newtonian doctrine of Absolute Space and Absolute Time. We will consider first his attempt to prove from the Principle of Sufficient Reason that there are not, never have been, and never will be two precisely similar material particles. The argument may be put as follows.

Suppose, if possible, that there are two co-existing material particles *A* and *B*, exactly alike in all their qualities and dispositional properties. They would have to be at different places. Suppose that *A* is at *P* and *B* at *Q*. For the present purpose it does not matter whether we assume the absolute or the relational theory of Space. If *P* and *Q* are points of Absolute Space there could be no possible reason for preferring to put *A* at *P* and *B* at *Q* rather than *B* at *P* and *A* at *Q*. But a similar consequence follows on the relational theory. In that case the point *P* is defined by certain spatial relations to a certain set of material particles chosen as a system of reference, and the point *Q* is defined by certain other spatial relations to the same set of material particles. Now, if the two particles *A* and *B* are precisely alike in all their qualities and dispositional properties, there can be no possible reason for preferring to put *A* into the former set of relations and *B* into the latter rather than doing the opposite with them. If, then, God were to create two such particles, he would (i) be bound to put them in different places, and yet (ii) would have no reason for choosing between the two alternatives which would arise by imagining the two par-

ticles transposed. Since God never acts without a sufficient reason, we can conclude that he never will create two precisely similar co-existing particles and therefore that there never will be two such particles.

The importance of this conclusion for the present purpose is the following. Leibniz used the Principle of Sufficient Reason as the basis of one of his main arguments against the theory of Absolute Space and Time. Suppose that the Absolute Theory of Space were true and that the material universe is of finite extent. Then it is intelligible to suggest that, without any difference in the mutual relations of various parts, the material universe as a whole might have been created by God in this, that, or another region of Absolute Space. But there could be no reason for preferring to create it in one region rather than in another. Therefore God would be faced with either (i) the alternative of not creating a material universe at all, or (ii) creating it in one or another of a number of alternative places between which he would have no possible ground for deciding. Since the material universe does in fact exist, we know that God has in fact created it. Since it is contrary to the nature of an intelligent being to make an unmotivated choice, we can be sure that God was not really faced with the alternatives which would have confronted him if the Absolute Theory of Space had been true. Now, if the Relational Theory were true, these so-called alternative ways of placing the world would not be genuine possibilities; for there could be no space prior to the existence of matter. On that theory God creates space in creating and arranging matter. So, Leibniz concludes, we can reject the Absolute Theory and accept the Relational Theory of Space.

A very similar argument can be used against Absolute Time. On the Absolute Theory it is intelligible to suggest that God might have created the world, with exactly the same contents and exactly the same subsequent history, at an earlier or a later moment of absolute time than that at which he in fact created it. Yet he could have no possible reason for preferring one moment to another at which to start the created world. The

argument then proceeds as before. On the Relational Theory of time these so-called alternatives do not exist; for time begins with the first event.

Now Clarke had answered by pointing out that God would be in precisely the same kind of difficulty on the Relational Theory if he created two exactly similar particles. And he assumed that Leibniz would admit that there might be, and in fact are, precisely similar particles, e. g., various atoms of the same substance. As we have seen, Leibniz's reaction was to accept Clarke's argument and to conclude that God would not create two precisely similar particles and therefore that the supposition that there are such particles may be rejected.

The logical position at this point is the following. Leibniz has tried to refute the Absolute Theory and support the Relational Theory by showing that a certain situation, which would conflict with the Principle of Sufficient Reason, *would* arise if the former theory were true and *could not* arise if the latter were true. Clarke counters this by saying that, if there are material particles which are precisely alike, a similar conflict with the Principle of Sufficient Reason will arise even on the Relational Theory; and concludes that Leibniz's argument cannot be decisive in favour of the latter. Leibniz counters this by accepting all Clarke's premisses except that there are precisely similar material particles, and concludes that God will never create such particles and that therefore there never will be such.

Clarke is not satisfied with this answer. He points out that a person might know that it would be much better to actualise *one or other* of two alternatives A and A' than to actualise *neither* of them, whilst at the same time he may see that it is a matter of complete indifference whether it should be A or A' that is realised. On Leibniz's principle a person in this position will realise *neither*, simply because he has no reason to prefer one to the other, although he has a very good reason for preferring to realise *one or other* of them to realising *neither*. Clarke says that in such a case of indifference a free agent chooses a certain one of the indifferent alternatives by a 'mere act of will'.

Leibniz answers that, if this were possible, which it is not, such motiveless choice would be indistinguishable from pure objective chance. I might remark that a man in this kind of situation would probably decide to associate the head of a coin with one of the alternatives and the tail with the other and to spin the coin and choose that alternative which was associated with the side that should fall uppermost. But this expedient would not be open to God; for he would know beforehand how the coin would fall, and so he would already be deciding on a certain alternative when he associated it with the face which he foresaw would fall uppermost.

I think that this part of Leibniz's argument might fairly be summarised as follows. Let us grant, for the sake of argument, that the Absolute Theory of Space and Time is in some sense an intelligible hypothesis and not just meaningless verbiage. If that theory were true the created universe could have occupied, without being in any way different internally, a different stretch of time or a different region of space. Now there would have been no possible reason for preferring to put it in one stretch of time or one region of space rather than another. Therefore God, who never makes a choice without a sufficient reason, would not have created a universe at all. But, since there is a universe, we know that he has created one. Therefore we can be certain that the Absolute Theory is false if it is not meaningless.

Like Clarke I find it hard to decide whether Leibniz would have gone further and said that the Absolute Theory is just meaningless verbiage. The following remarks in his Fifth Letter are typical. The supposition of the universe as a whole being moved is (i) meaningless, since there is no space outside it; and (ii) even if it were intelligible it would be pointless for 'there would happen no change which could be observed by any person whatever'. He adds the following remark: 'Mere mathematicians, who are only taken up with the conceits of imagination are apt to forge such notions, but they are destroyed by superior reasons.' In the same Letter he says that real change must be *in principle*

observable. Motion need not be actually observed; but there is no motion where there is no change that *could be* observed, and there is no change where none *could be* observed.

All this has a very modern ring, and might have been said by any contemporary Logical Positivist. Nevertheless I do not feel quite sure how to interpret it. It seems to me that it is fairly susceptible of either of the following two interpretations. (i) The Absolute Theory, and various questions which arise in connexion with it, are *intrinsically* meaningless; and so we must accept the Relational Theory. (ii) Even though the Absolute Theory be *not intrinsically* meaningless, and though these questions be intelligible in terms of it, yet we can reject it and accept the Relational Theory because of the argument founded on the Principle of Sufficient Reason. And *in terms of the Relational Theory* these questions *are* meaningless. I am inclined to suspect that Leibniz himself held the first view, but contented himself with the second for controversial purposes.

It is now convenient to give Leibniz's positive account of what is meant by 'Space' and 'Time'. He introduces this topic in the Fourth Letter and goes into considerable detail about Space in the Fifth. He does not discuss Time in similar detail; no doubt he thought, as so many writers on these topics have done, that what holds for Space can be applied automatically to Time. This is, in my opinion, a dangerous assumption, for the unlikenesses are at least as important as the likenesses.

According to Leibniz, Space is an order of coexistences and Time is an order of sequences. This seems to me plainly inadequate; for events may be contemporary as well as successive, and we can give no account of either rest or motion unless we can conceive of identity of place at different times as well as difference of place at the same time. However, Leibniz is fully aware of the latter point and deals explicitly with it in the full account of Space which he gives in the Fifth Letter. This may be summarised as follows.

Suppose that certain bodies, X, Y, Z . . . , etc., do not change their mutual spatial relations during a certain interval. Suppose

further that, if there is a change during this interval in their spatial relations to certain other bodies, then the cause of it has not been in themselves. Then we can say that the bodies *X*, *Y*, *Z* . . ., etc., have constituted a 'rigid fixed system' during the interval in question. Suppose that, at some moment within this period, a certain body *A* stood in certain spatial relations to the bodies of this system; that at a later moment within the period it ceased to stand in those relations to them; and that at some later moment within the period a certain other body *B* began to stand to those bodies in precisely similar relations to those in which *A* had formerly stood. Then we can say that '*B* had come to *occupy the same place as A* formerly occupied'. If and only if the cause of these changes of relative position have been in *A* and in *B*, we can say that *A* and *B* have 'been in motion'.

Leibniz then defines 'a place' in terms of the relation of 'occupying the same place', and he defines 'space' as the collection of all places. He makes several interesting comments on this procedure.

(1) He remarks that, in making the notion of sameness of place primary and defining 'place' in terms of it, he follows the procedure of Euclid who starts by defining the statement that *A* has *the same ratio* to *B* as *C* has to *D* and does not begin by defining 'ratio'.

(2) He remarks that, if *B* occupies the same place as *A* did, we must not say that the present relation of *B* to the system of reference is *the same* as the previous relation of *A* to that system. Two different things cannot literally stand in the same relationship. We must say that the relationships are precisely alike. He then adds the following remark. "The mind, not contented with an agreement, looks for an identity — for something that should be truly the same; and conceives it as being extrinsic to the subject; and this is what we here call 'place' and 'space'. But this can only be an ideal thing; containing a certain *order* wherein the mind conceives the application of relations."

The upshot of the matter is this. Speaking in the terminology of contemporary Cambridge logicians we may say that Leibniz

regards Space as a logical construction out of places, and regards a place as a logical construction out of facts about relative spatial position. And he holds that the notion of Absolute Space and absolute places is a fallacy of misplaced concreteness.

It will be noticed that, in defining 'sameness of place at different times', Leibniz has had to introduce the condition that the system of reference shall not have moved during the interval. It will also be noticed that he gives a causal criterion for judging whether a body or system of bodies which has changed its relative position shall be said to have moved or not. The criterion is whether the cause of the change of relative position is or is not in the body itself. He uses this criterion in order to answer Newton's empirical arguments for absolute rotation based on the existence of centrifugal forces. In reference to this argument he says that there is nothing in it that proves the reality of Absolute Space. There is a difference, even on the Relational Theory, between what he calls 'an absolute true motion of a body' and what he calls 'mere relative change of its situation with respect to another body'. But this difference, he says, consists in whether 'the immediate cause' of the change of relative position 'is in the body itself' or not. I take it that his view is that centrifugal forces are connected with 'absolute true motions' thus defined.

It seems to me that a *prima facie* objection to this criterion is that, according to the First Law of Motion, the cause of an accelerated or a curvilinear motion of a body never is in that body itself. Leibniz would not have accepted this objection because he had a general metaphysical principle that all the changes in any substance are caused by its own previous states, and that the appearance of interaction between different substances is delusive. He attempts to construct a system of dynamics in accordance with that principle, but it would take us outside our present limits to discuss it.

We can now pass to another point in the controversy. Clarke in his Third Letter said that space and time are magnitudes, whilst order and situation are not, and he made this an objec-

tion to the relational theory. He reiterates this objection in his Fourth Letter and complains that Leibniz has made no attempt to answer it. Leibniz deals with the objection in his Fifth Letter. He says there that relations can have magnitude. The examples which he gives are ratios between numbers. E. g., the ratio of 28 to 7 is equal to the ratio of 16 to 4 and is greater than that of 15 to 5. Now ratios are relations, and any pair of terms of which it is intelligible to say that one is equal to, greater than, or less than another is a magnitude. He adds that the magnitudes of ratios are measured by their logarithms. I suppose that this is because, if the ratio of x to y is l and that of y to z is m and that of x to z is n , then $\log l + \log m = \log n$. Clarke answers that ratios are not magnitudes because they are not additive. I think that the point is that $x/y + y/z$ is not in general equal to x/z . He considers the reference to the additive property of logarithms irrelevant. And, in any case, he says, time and space are not of the nature of ratios. They are absolute magnitudes which have ratios among themselves.

It seems to me that the questions at issue are confused by Leibniz's reference to the example of ratios in arithmetic. Presumably the fundamental relations on the Relational Theory are (i) the relation of *distance* between two material particles, and (ii) that of *angular divergence* between the lines joining one pair of material particles and another pair. I see no objection to saying that these are magnitudes. In certain special cases, viz., if three particles x , y , and z are collinear and y is between x and z , the distance between x and z is the sum of the distances between x and y and between y and z . But in general the relationship is more complex. Similar remarks, *mutatis mutandis*, apply to angular divergence between lines. So far Leibniz seems to be in the right. On the other hand, we have also to consider area and volume; and Clarke seems to be right in calling these absolute magnitudes which have ratios among themselves. But I do not think that this would be any reason for accepting the Absolute Theory of Space and rejecting the Relational Theory,

though it might show that the Relational Theory needs certain supplements.

The next point that I shall consider is the question of the finitude or infinitude of the material world in space and of whether it is a plenum or contains empty regions within it. The Newtonians held that the material world is of finite extent and that outside it there is a boundless expanse of Absolute Space. They also held that within the universe there are regions of Absolute Space which contain no matter. Leibniz denied both these propositions. According to him the material universe continuously occupies an unlimited expanse.

The details of the controversy are rather tedious, so I shall try to state briefly in my own way what I believe to be the facts of the case. (1) The only alternatives among those just mentioned which would have a meaning on the Absolute Theory (assuming that that theory is itself intelligible) and would be meaningless on the Relational Theory are the following. (i) That the universe as a whole should rotate or not rotate about an axis. (ii) That, if the universe be finite in extent, it should as a whole either have a motion of translation or be translationally at rest.

(2) On the Relational Theory it is *prima facie* intelligible that the universe should be either finite or infinite in extent. The former alternative would mean that, if you take the distance between any two particles P and Q as your unit, then there is a finite integer N such that the distance between any two particles in the universe is less than N times the distance between P and Q . The latter alternative would mean that, if you take the distance between any two particles P and Q as your unit and measure in any direction from any assigned particle O , then, whatever finite integer N may be, there is always a particle in that direction at a greater distance from O than N times the distance between P and Q . (I call this *prima facie* intelligible, because it involves no internal contradiction. Whether this kind of actual infinity be not unintelligible in some *other* important sense is a question which I cannot discuss here.)

(3) If the universe is of finite extent, it is intelligible on the Relational Theory to say (i) that it *might have been* bigger or smaller at a given moment than it in fact was then, and (ii) that it might *become* bigger or smaller in future than it now is. For this means simply that the finite integer N , mentioned above in the definition of the finitude of the universe, might have been bigger or smaller than it in fact was or might become bigger or smaller than it now is.

(4) On the Relational Theory it is equally intelligible to suggest that matter is continuous or that there are holes in it. We could define an 'empty linear segment' as a pair of particles P and Q such that there was no particle between them. Having done this we should have no difficulty in principle, though there would be considerable difficulties in detail, in defining an 'empty area' and an 'empty volume'.

It seems to me then that there is no close logical connexion between the controversy about Absolute and Relative Space, on the one hand, and these controversies about the finite or infinite extent of the universe and the existence or non-existence of empty spaces within it, on the other. In the end Leibniz says explicitly that he does not maintain either that God *could not* have limited the quantity of matter or that he *certainly has not* done so. He asserts only that it is *very unlikely* that a perfectly wise and benevolent creator would have done so. This is his position in the latter part of the Fifth Letter. But earlier in the same Letter he uses phrases which suggest that the Relational Theory suffices to settle the question in favour of the infinity and continuity of matter. He says: 'Since space in itself is an ideal thing . . . space outside the world must needs be imaginary . . . The case is the same with empty space within the world, which I take also to be imaginary . . .' Immediately after this passage he goes on to discuss the allegation that Guerike of Magdeburg had produced a vacuum in the receiver of his air-pump.

If Leibniz meant merely that, on the Relational Theory, space does not exist, in the sense in which the Newtonians thought it did, either outside the material universe, if that be finite, or

inside the receiver of an air-pump, he was no doubt right. But, if he thought that this has any tendency to prove that the material universe cannot be finite in extent and cannot have empty holes in it, he was, as I hope I have shown, quite mistaken.

I have now stated the arguments which Leibniz used and the conclusions which he drew in his *Letters to Clarke* about the absolute and relational theories of Space and Time. But it is important to remember that this controversy is conducted at what Leibniz would regard as an intermediate level of philosophical rigour and thoroughness. It is indeed a philosophical, and not merely a physical, discussion. But in it Leibniz is granting for the sake of argument certain assumptions which he would claim to have refuted in his more elaborate and professional philosophical writings. He is granting here the reality of extended substances and of spatial relations between them, but in fact he believes himself to have shown that the notion of an extended substance involves a contradiction, and that there can be no relations between substances. According to his considered opinion, what we misperceive as an endlessly divisible extended material thing is really a collection of an infinite number of unextended mental substances, and what we misperceive as a relation between two things is really certain qualities in the things which we misperceive as interrelated. I shall end this paper by showing that, at this deeper level, Leibniz's view is in an important sense a form of the absolute theory.

In order to do this I must first explain a distinction which was originally pointed out by the Cambridge philosopher, W. E. Johnson, in his *Logic*. I shall put it in my own way. In controversies about the absolute *versus* the relational theory of space and time there are two questions to be distinguished: — (1) Is position a pure quality or a relational property? (2) Does position belong to material particles *directly*; or does it belong primarily to particulars of another kind, viz. points of space, and only in a derivative sense to material particles in virtue of their occupation of points of space? The first question may properly be put in the form: 'Is spatial position qualitative or

relational? 'The second may properly be put in the form: 'Is space adjectival or substantival in character?' Johnson pointed out, quite rightly in my opinion, that these two questions were never clearly distinguished by protagonists in the controversy about 'absolute' *versus* 'relative' space.

We can begin by dividing possible theories into (1) Substantival, and (2) Adjectival. The essential features of all forms of the Substantival Theory are the following. There are particulars which together constitute a single complex particular, viz., Space. These and only these have spatial characteristics in the primary and underived sense. And each of them has timelessly or sempiternally all the spatial qualities and relational properties that it has. It is meaningless to talk of a point of Space, in this sense, changing its position; or of a volume of Space, in this sense, changing its size or shape. Now, besides Space and its regions or points, there are material things or particles. Each material particle at any moment occupies a certain point of Space, and each body at any moment occupies a certain region of Space. At different moments the same material particle or body may occupy the same or different points or regions of Space. The statement that a certain body has at a certain moment a certain position, shape, size, etc., is always derivative and analysable. It means that at this moment that body occupies a region of space which timelessly or sempiternally has a certain position, shape, size, etc. A body can change in respect of its spatial characteristics because (i) it can occupy different regions of Space at different times, and (ii) these regions must differ timelessly or sempiternally in position and may differ timelessly or sempiternally in shape and size.

The essential features of all forms of the Adjectival Theory are the following. The only subjects of spatial characteristics are material particles or bodies. There is not another kind of particular existent called 'Space' beside matter. The spatial characteristics which a material particle or a body has at any moment belong to it in a primary and underived sense. So there are no timeless or sempiternal spatial characteristics. A body may hap-

pen to keep the same position, shape, and size for a long time, or it may happen to change quickly and continuously in respect of some or all of these characteristics. But there can be no question of analysing such a change into a relation of occupance to a series of terms of an entirely different kind, each of which has all its spatial characteristics timelessly or sempiternally.

I think there is no doubt that the Newtonians held, and that Leibniz rejected at all levels of his thinking, the *substantial* theory of Space and Time.

We can now consider the other pair of opposites, viz., (1) Qualitative and (2) Relational, theories. We will confine ourselves here to the characteristic of spatial position, and not consider shape or size. I think that the Qualitative Theory may be put most clearly as follows. There is a certain determinable quality, which we will call 'Spatial Position'. We might compare this to the determinable Sound-quality. The determinates under it form a continuous three-dimensional manifold of qualities. These may be compared (though the analogy must not be pressed in detail) with the manifold of determinate sound-qualities which can be arranged in respect of pitch, loudness, and timbre. Any two particulars which have simultaneously two different determinate forms of the determinable quality of Spatial Position will *ipso facto* stand to each other in certain determinate relations of distance, direction, etc. This may be compared with the fact that any two sounds which have simultaneously two different determinate forms of the determinable Sound-quality will *ipso facto* stand to each other in certain relations of harmony or disharmony, of relative loudness, and so on. Thus spatial relationships are *founded upon* the determinate positional qualities of the related terms; just as musical relationships are founded upon the determinate sound-qualities of the notes struck.

The essential features of the Relational Theory are the following. There is no *quality* of spatial position. The fundamental positional characteristics of any term are its *relations* of distance and direction to other terms. These relations are not founded upon qualities in the related terms, as the musical

relations between notes are founded upon their determinate sound-qualities. To say that a certain particular has a certain position is simply to state its relations of distance and direction to certain other particulars of the same kind chosen arbitrarily as terms of reference.

Now it would be possible theoretically to combine either the Substantival or the Adjectival theory with either the Qualitative or the Relational theory. But in fact the usual combinations have been Substantival Qualitative and Adjectival Relational. I think there is little doubt that the Newtonians held a form of the Substantival Qualitative theory, and it is certain that Leibniz in the letters to Clarke is asserting a form of the Adjectival Relational theory.

But this is not the theory that Leibniz really held. What he really held, when he was not arguing *ad hominem* against Clarke and the Newtonians, was a form of Adjectival Qualitative theory. That this must be so is obvious in view of his general principle that what we take to be relations between terms are really qualities in the terms which we partially misperceive. But we need not confine ourselves to such general considerations; for Leibniz has told us explicitly what is the determinable quality, present in some determinate form in every monad at every moment, which is the basis of the appearance of spatial relations. It is what he calls 'Point of View'. It is true that he would not allow us to identify point of view with the quality of spatial position; but point of view is a quality, and every difference in the apparent spatial position of apparently extended objects is correlated with a real difference in the point of view of the monads which we misperceive as those extended objects.

Suppose that we were to drop the distinction between the quality of Point of View and the quality of Spatial Position, and to speak wholly in terms of the latter. Then the Adjectival Qualitative theory of spatial position could be formulated as follows. There is a determinable quality of Spatial Position, and under it there is a three-dimensional manifold of determinate positional qualities. At each moment each material particle has

one and only one of these. At two different moments the same material particle may have the same or different determinate positional qualities. At any moment any two material particles will stand in a determinate relation of relative position, which is founded upon the determinate positional qualities possessed at that moment by each of them. Absolute motion of a particle consists in its having, at each of a continuous series of moments, a different one of a continuous series of positional qualities. Relative motion of one material particle with respect to another entails that at least one of them is in absolute motion; but the same relative motion could arise in connexion with very different absolute motions of the two particles concerned. It seems to me that this is the kind of view of space and motion which we ought to ascribe to Leibniz when we dig beneath the position which he occupies in his controversy with Clarke.
